# Auction Theory: Strategies, Models and Applications

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**Abstract**

*The paper seeks to explain auction theories, strategies and their applications particularly in floating tenders. It starts by explaining the different models that have existed. It also talks about the malpractices that are possible in this process and how tenders can be detrimental for the PSUs. The paper also touches upon how the Jan Aushadhi scheme of the government has effectively used the tender system to procure best quality drugs at the lowest prices. After studying a few cases, we come to the conclusion that closed dutch auctions are the most effective when it comes to receiving most effective and efficient bids.*

## Introduction

Unlike large parts of economics, the real-world applications of auctions are considered prior to their theoretical basis. This branch of economics examines the various strategies that competing parties can adopt in their attempt to emerge successful from the auction. Contrary to popular belief, there exist multiple auction formats, each of which have their own unique bidding strategy.

This paper seeks to collate these formats and strategies, while discussing in brief he fields in which they are applied. The main focus of this paper will be to analyze how effectively closed Dutch Auctions are used by the Indian Government when they float tenders. However, while explaining the various auction modalities, different real-world examples world be used, ranging from the manner in which sports players are auctioned to the manner in which paintings and artifacts are auctioned. These help make the model more lucid.

The aforementioned tenders relate to the maintenance or creation of public sector infrastructure, which the government is either overburdened or incapable of doing. By floating tenders, several firms are allowed to quote prices at which they will carry out the given task, and the firm that proposes the lowest sum receives the rights to that particular task. The paper will assess whether or not floating tenders is the most efficient allocation of resources, and recommend suggestions to existing processes to increase government efficiency as well as private-sector participation. The ubiquity of the tendering process is one that has not been probed into adequately, and while it does appear efficient, it may have its faults, especially when applied to a country with a political and economic climate such as India.

The paper will open by examining existing models and research on auctions, both game theoretic as well as probabilistic. The paper will then use a simple matrix to explain and term different auction formats that will be used frequently through the paper. It will then examine each model separately, and use simple game theoretic models to further elucidate the same. Whenever possible, matrices and the like will be used. The paper will conclude by examining the energy sector in India, in all it’s tendering processes, pays heed to strategies that have been delineated. The paper will also qualitatively examine how, and to what extent, auctions provide efficient outcomes in practice and whether there can be rudimentary adjustments made to increase the government's benefit in this process while retaining the attractive, competitive nature of auctions.

From the book *Networks, Crowds, and Markets: Reasoning about a Highly Connected World*, by David Easley and Jon Kleinberg. Cambridge University Press, 2010, basic jargon and models can be developed. This chapter of the book is comprehensive and addresses most of the auction formats and strategies that this paper aims to cover, however it represents this in a cumbersome manner and does not relate the same to the real-world. It does however explain the theory behind when auctions can be a useful tool. The concepts of revenue equivalence and the seller's point of view have also been described in detail, creating a comprehensive reference point for much of this paper.
However is Jonathan Levin's 2004 paper that delves in to great detail regarding the game theoretic nature of this agenda.

## Types of Auctions

1. Ascending-Bid Auctions

These are also called English auctions. These auctions are carried out interactively in real time, with bidders present either physically or electronically. The seller gradually raises the price, bidders drop out until finally only one bidder remains, and that bidder wins the object at this final price. Oral auctions in which bidders shout out prices, or submit them electronically, are forms of ascending-bid auctions (Levin, 2004).

In order to simplify and develop this model, we assume the following scenario:

An Item I is being auctioned to i bidders. This auction is independent of any prior or succeeding auctions. Bidders $_{B1}$, $B_2$, $B_3$……Bi have pre-determined values $V_1$, $V_2$, $V_3$…….$V_i$ which have been computed by each bidder, referred to as signals. These values are unknown and independent of the values of other Bidders. Each Bidder ($_{B1}$, $B_2$…..$B_i$) submits bids $_{b1}$, $b_2$, $b_3$…..$b_i$ such that $b_n > b_{n-1}$. As the auction proceeds, Bidders drop out if they are unwilling to up the current bid. This continues until a single bidder remains who will pay the current bid.

It is important to determine a link between individual bids and their signals. It is this link that will guide bidders in determining their strategies when put in the interactive context of an English auction.

In the case of English auctions, a player's dominant strategy is that the highest bid will match ones value. Assume the following proposition:

$_{B1}$, with signal s1 has submitted his previous bid $_{b1}$. There exist three possible scenarios:

i. $_{b1} < s_1$

If $_{B1}$ wins the auction at $_{b1}$, he would have a surplus of $s_1 - {}_{b1}$ – a situation beneficial to him as there exists an imputed gain.

ii. $_{b1} = s_1$

In this scenario, the buyer is at an equilibrium as there his value equals the price he has bid. Winning the auction at this price would neither create a surplus nor a deficit for $_{B1}$.

iii. $_{b1} > s_1$

This would imply that the previous submitted bid $_{b1}$ exceeds the value assigned to the item ($s_1$) by $_{b1} - s_1$, which results in an imputed loss of $_{b1} - s_1$. This would be inefficient and irrational in a Marshallian sense, as marginal utility and price are not in congruence.

In the first scenario, if bidders $B_2 \ldots\ldots B_i$ made a bid higher than $_{b1}$, $_{B1}$ should mirror such movements up until the bid reaches $s_1$. When $_{b1} = s_1$, the micro-market is at equilibrium.

If after bidding $s_1$ there comes a higher bid, it is irrational for $_{B1}$ to match any subsequent bids as there would be an imputed loss.

It is therefore clear that the dominant strategy in English auctions is to bid ones value.

More specifically it is a weakly dominant strategy. It provides at least the same utility to all bidders (zero, as a single bidder succeeds in securing the item) and greater than zero for the bidder that values the item the highest. For the specific bidder, this is a strictly dominant strategy.

2. Descending-bid auctions

This format is also called a Dutch auction. This is also an interactive auction format, in which the seller gradually lowers the price from some high initial value until the first moment when some bidder accepts and pays the current price. These auctions are called Dutch auctions because flowers have long been sold in the Netherlands using this procedure.

Dutch auctions can be modeled in the same manner as an English auction barring the section regarding bids, as there exists just a single bid. The three scenarios still apply, and it is clear that by bidding higher than the value, there is a loss that arises.

The difference between Dutch and English auctions arise when bidders look at creating some sort of consumer surplus ($b_1 < v_1$). In English auctions, there is a margin for error as if other bidders decide to enter the auction and raise the price, the bidder in question has the opportunity to match these bids or drop out. However, in Dutch auctions since there exists just the single bid, by waiting for the price to drop below $v_1$, to say $b_*$, the bidder runs the risk of losing out on the item if another player values the item anywhere between $v_1$ and $b_*$ and adopts the dominant strategy of bidding as per his/ her value. It can argued along these lines that the lack of breathing space in a Dutch auction can make for a more efficient form of resource allocation, and is likely to result in stricter alignment with the strategy of paying ones value. This makes the generation of profits somewhat limited in scope.

3. First-price sealed-bid auctions

In this kind of auction, bidders submit simultaneous “sealed bids” to the seller. The terminology comes from the original format for such auctions, in which bids were written down and provided in sealed envelopes to the seller, who would then open them all together. The highest bidder wins the object and pays the value of her bid.

Unlike Dutch and English auctions, there is no information that can be drawn regarding the responsiveness of bidders to price movements. This complete opacity in information results in bidders adhering strictly to strategies revolving around their own value calculations.

This greater allegiance to one’s own value implies, to a certain extent, that the possibility of profit generation is decreased.

This can be modeled as follows:

An Item I is being auctioned to i bidders. This auction is independent of any prior or succeeding auctions. Bidders $B_1$, $B_2$, $B_3$……$B_i$ have pre-determined values $V_1$, $V_2$, $V_3$…….$V_i$ which have been computed by each bidder, referred to as signals. These values are unknown and independent of the values of other Bidders. Each Bidder ($B_1$, $B_2$…..$B_i$) submits a single bid b. All the bids are collected, and the bidder whose bid was the highest receives the item.

The bidder can bid in three manners, with b being less than, equal to, or greater than $v_1$. As is the case with Dutch auctions and English auctions, bidding higher than ones value is irrational, as successfully obtaining the object through this bid will lead to a loss. Bidding lesser than ones value is, as is the case with the Dutch auction, risky as there is once again only a single round of bidding. Although this is the only way that bidders can earn a profit, has elements of great uncertainty as the responsiveness to price movements is completely unknown. Bidding at ones value ensures that in the event that the bid is the highest, there is no possibility of a loss. This also removes any possibility of profit.

4. Second-price sealed-bid auctions, also called Vickrey auctions.

Bidders submit simultaneous sealed bids to the sellers; the highest bidder wins the object and pays the value of the second-highest bid. These auctions are called Vickrey auctions in honor of William Vickrey, who wrote the first game-theoretic analysis of auctions (including the second-price auction). Vickery won the Nobel Memorial Prize in Economics in 1996 for this body of work.

Assume the following scenario:

An Item I is being auctioned to i bidders. This auction is independent of any prior or succeeding auctions. Bidders $B_1$, $B_2$, $B_3$……$B_i$ have pre-determined values $V_1$, $V_2$, $V_3$…….$V_i$ which have been computed by each bidder, referred to as signals. These values are unknown and independent of the values of other Bidders. Each Bidder ($B_1$, $B_2$…..$B_i$) submits bids $b_1$, $b_2$, $b_3$…..$b_i$ such that

$b_n > b_{n-1}$. As the auction proceeds, Bidders drop out if they are unwilling to up the current bid. This continues until a single bidder remains who will pay an amount equal to that paid by the second to last bidder.

We will once again hypothesize that bidding according to one's value is the dominant strategy. Let b refer to the current highest bid. There exist three scenarios:

i. $b > v_1, b_1$

In this scenario, $_{B1}$ will not win the auction as the current bid exceeds the amount $B_1$ is willing to pay according to her value.

ii. $v_1 < b < b_1$

If $_{B1}$ bids above their value ($b_1 > v_1$) and wins the auction with the bid of $b_1$, there is a loss of $b - v_1$. $B_1$ will have to pay b as it is the second to last price that was bid. This price, b, is still higher than the value $v_1$.

iii. $b < v_1, b_1$

If $B_1$'s bid of $v_1$ is the highest bid, he will have won the auction and will pay a price of b (the second to last price), implying that his surplus is $v_1 - b$. This will occur when $b_1$ and $v_1$ are both above b and are congruent with one another.

Auctions are generally used by sellers in situations where they do not have a good estimate of the buyers' true values for an item, and where buyers do not know each other's values. In this case, as we will see, some of the main auction formats can be used to elicit bids from buyers that reveal these values.

## Auctions as a Pricing Mechanism

To motivate the setting in which buyers' true values are unknown, let's start by considering the case in which the seller and buyers know each other's values for an item, and argue that an auction is unnecessary in this scenario. In particular, suppose that a seller is trying to sell an item that he values at x, and suppose that the maximum value held by a potential buyer of the item is some larger number y. In this case, we say there is a surplus of $y - x$ that can be generated by the sale of the item: it can go from someone who values it less ($x$) to someone who values it more ($y$).

Assume the following scenario:

The seller values the item at $x$, and out of the potential buyers, there exists one that values the item at $y$, where $y>x$. In the event that such a sale goes through, we see the transfer of an item from the buyer who values it at $y$ to the seller who values it at $x$, who earns a surplus of $y – x$.

The premise of using auctions as a pricing mechanism lies in an asymmetry of information between buyers and sellers. Consider the contrary – that buyers and sellers are both aware of the value that they each assign to the item at hand. In this case, the seller can simply announce that the item is for sale at a fixed price just below $y$, and that he will not accept any lower price. In this case, the buyer with value $y$ will buy the item, and the full value of the surplus will go to the seller. In other words, the seller has no need for an auction in this case: he earns a considerable surplus just by announcing the right price.

There are few situations where such symmetry of information exists and the pricing mechanism lies wholly in the control of the seller. This implies that the sellers decision on price is the only one that is regarded. As was seen in the above case, when the seller is given the right to "tie his hands" and commit to a fixed price, he is the sole recipient of any surplus that arises ($y – x$). This is an inevitable situation in auction-less cases – the pricing mechanism is biased toward the seller who is able to dictate prices.

In the event of an auction, the responsibility of price determination lies with the buyers. Barring the 'base price' set at auctions by the sellers, any other movements in price are purely determined by the buyers. If, as the auction proceeds, the price finally arrived at is $z$ (where $y > z > x$), we can see a market less biased toward sellers. This is because the surplus that can be earned by the seller is reduced to ($z – x$), and there is a surplus generated for the buyers ($y – z$), when compared to the seller-dictated mechanism.

## Common Value Auctions

Bidding strategies and models have been developed, in the above sections, for auctions for items being transferred from sellers to consumers, with no intention of being resold. The previous cases were typified by the complete independence of value amongst buyers. In practice, this may not be the case. In several situations, especially in the case of tenders and the like, the buyers are acquiring rights to something – be it drilling rights or bandwidths. The value of the final product, such as the value of the oil under the plot of land being auctioned, can be estimated by the potential bidders, each of which will be privy to some information regarding the item which will color their estimates. This generates a 'common value' which is private yet is likely to be similar to other estimates.

To model auctions around the existence of a common value, we must estimate the value of the item in context of the estimated common value. This can be done as –

$v_i = v + x_i$

where v is the value of the item, and x is the error component that is bound to exist in each of the several estimates of the common value.

While the flow of common value auctions is identical to all other auctions, it is in the analysis of the result where this format differs. If all bidders bet according to their value, as modeled by the aforementioned function, the eventual winner of the auction will have estimated the common value far greater than any other competitor. This overestimation can take place in any auction format. This question arises as a ballpark figure of the resale value is known. This phenomenon, where the winner tends to overpay for an item, is known as the Winner's Curse.

This is known as the winner's curse, and it is a phenomenon that has a rich history in the study of auctions. Such scenarios take place in the pharmaceutical industry, when patents are being auctioned, or in the case of football players – where the common value is the performance of the player.

Rational bidders must take the winner's curse into account while bidding, and the amount bid must be a combination of the private estimate of the common value as well as competitive enough to win the object. Bidders are likely to 'shade' their bids downward in order to compensate for winner's curse creeping in. This will tend to be lower than the value assigned, thus establishing that when there is an increase in information regarding the value of the item, bidders tend to be far more cautious in their approach.

## The Tender System

The Government of India, by its nature of being, is obligated to have unbiased, impartial and fair mechanisms while allotting work that is directly or indirectly paid for by the taxpayers of the country. A tender system may seem foolproof as the work is rewarded purely on the basis the efficiency and economy of the organization seeking work, the lowest bid wins the job. Kickbacks have however inspired and motivated people to find their way around this seemingly pure system.

After 'The Greatest Auction Ever' of 1995, where the telecommunications spectrum was auctioned, most countries that were under the influence of the World Bank and the IMF began to use the tender process for government work because it allows free and unfettered competition. It celebrates efficiency and grants a certain degree of accountability as to where the money goes. The British spectrum auction of 2000, which raised about $34 billion, earned one of its academic designers 5 a commendation from the Queen and the title "Commander of the British Empire. " In the same period, game theorists were plying their trade on another important application as well. The National Resident Matching Program, by which 20,000 US physicians are matched annually to hospital residency programs, implemented a new design in 1998 with the help of the economist – game theorist Alvin Roth.

Before a tender is floated, a few conditions are agreed upon by the organization. This includes

- quality and safety standards
- quantity is specified to avoid excess
- Reasonableness of rate

To ensure a transparent procedure, it is essential that the tender is adequately advertised such that eligible parties know about it. After receiving the bids, the government checks whether the bidders adhere to the standards and carry out a prudential check on the price quoted by the bidder. If the tenderer deems the company fit, a proposal is drawn.

## The Tendering process in the Pharmaceutical Industry

Pharmaceutical products fall under the ambit of necessities. Several state governments and bodies of the union government float tenders to meet hospital and PDS requirements.

Under the Jan Aushadhi scheme of the Indian government, the government has managed to supply medicines at prices as low as Rs. 3.81 for 10 tablets of paracetamol. The tender comes with a lot of time based conditions. Failure to comply with these may result in blacklisting that does not allow you to participate in subsequent tenders and imposes a penalty in lieu of the inconvenience caused.

Confidentiality in terms of other bids helps in ensuring that each company gives the lowest price possible. It also ensures that the companies don't indulge in malpractices by tweaking their prices by insignificant miniscule amounts to spite the best bidder.

The bankruptcy formula:

A very common dirty practice in the tender business is declaration of bankruptcy after winning the tender.

For example, a company bids an unrealistically low price to win the tender. They will get the tender by the virtue of being the lowest bidder. However, a sane mind may dictate that it is not possible for such a product to be made in low values. This company may offer incentives to the officials and get this tender. Since, it is not possible to carry out the consignment; the company declares bankruptcy thereby shedding all liability. The advance payments are then divided amongst the people.

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